Open Problems in L_p-Testing

See our joint work with P. Berman and S. Raskhodnikova (STOC'14).

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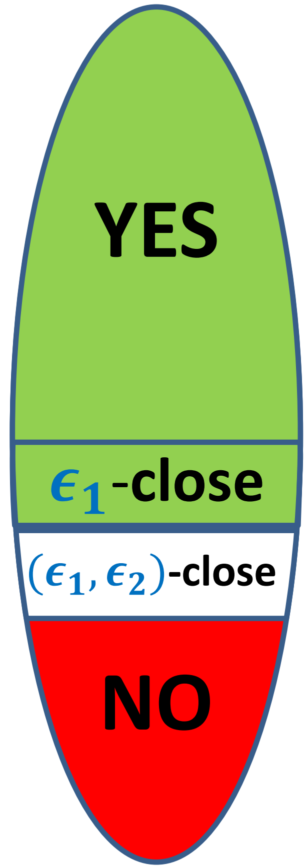
Tolerant "L_1 Property Testing"

- $\boldsymbol{f}: \{1, \ldots, n\}^d \rightarrow [0,1]$
- $\boldsymbol{P}$ = class of functions (monotone, convex, Lipschitz)
- $dist_1(\boldsymbol{f}, \boldsymbol{P}) = \dfrac{\min_{\boldsymbol{g} \in \boldsymbol{P}} |\boldsymbol{f} - \boldsymbol{g}|_1}{n^d}$
- $\boldsymbol{\epsilon}$-close: $dist_1(\boldsymbol{f}, \boldsymbol{P}) \leq \boldsymbol{\epsilon}$
- $\epsilon_1 = 0$ => Non-tolerant
- $\epsilon_1 \neq 0$ => Tolerant

Tolerant "L_1 Property Tester"



$\Rightarrow$ **Accept with probability $\geq \frac{2}{3}$**

$\Rightarrow$ **Don't care**

$\Rightarrow$ **Reject with probability $\geq \frac{2}{3}$**

Known non-tolerant L_1-Testers

- Monotonicity: $\boldsymbol{f}: [\boldsymbol{n}]^{\boldsymbol{d}} \to [0,1]$:

  $O\left(\frac{\boldsymbol{d}}{\boldsymbol{\epsilon}} \log \frac{\boldsymbol{d}}{\boldsymbol{\epsilon}}\right)$ (see BRY'14 for lower bounds)

- Lipschitz property $\boldsymbol{f}: [\boldsymbol{n}]^{\boldsymbol{d}} \to [0,1]$:

  $\Theta\left(\frac{\boldsymbol{d}}{\epsilon}\right)$ (tight)

- Convexity $\boldsymbol{f}: [\boldsymbol{n}]^{\boldsymbol{d}} \to [0,1]$:

  $O\left(\boldsymbol{\epsilon}^{-\frac{\boldsymbol{d}}{2}} + \frac{1}{\boldsymbol{\epsilon}}\right)$ (tight for $\boldsymbol{d} \leq 2$)

- Submodularity $\boldsymbol{f}: \{0,1\}^{\boldsymbol{d}} \to [0,1]$

  $2^{\tilde{O}\left(\frac{1}{\epsilon}\right)} + poly\left(\frac{1}{\boldsymbol{\epsilon}}\right) \log \boldsymbol{d}$ [Feldman, Vondrak 13, …]

Open Problem #1

- Complexity for non-tolerant L_1-testing convexity grows exponentially with d

Is there an L_1-testing algorithm for convexity with subexponential dependence on the dimension?

- Why is it hard?
  - Relevant reference: [Rademacher, Vempala'04]
  - Restrictions on 1-dimensional axis-parallel lines don't help (need exponentialy many)
  - Can 2-dimensional restrictions help?

L_1-Testing for Convex Optimization

- **Theory:** Convergence rates of gradient descent methods depends on:
  - Convexity / strong convexity constant
  - Lipschitz constant of the derivative
- **Practice:**
  - Q: How to pick learning rate in ML packages?
  - A: Set 0.01 and hope it converges fast
- Even non-tolerant L_1-testers can be used to sanity check convexity/Lipschitzness

Known tolerant L_1-testers

- Monotonicity in 1D

$$O\left(\frac{\boldsymbol{\varepsilon_2}}{(\boldsymbol{\varepsilon_2}-\boldsymbol{\varepsilon_1})^{\mathbf{2}}}\right)$$

- Monotonicity in 2D

$$\tilde{O}\left(\frac{\mathbf{1}}{(\boldsymbol{\varepsilon_2}-\boldsymbol{\varepsilon_1})^{\mathbf{4}}}\right)$$

Open Problem #2

- Only have tolerant monotonicity for $d = 1,2$.

Tolerant testers for higher dimensions?